

Deducing rest masses of quarks with a three step quantization

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Abstract. Using a three step quantization and phenomenological formulae, we can deduce the rest masses and intrinsic quantum numbers (I, S, C, B and Q) of quarks from only one unflavored elementary quark family ϵ with $S = C = B = 0$ in the vacuum. Then using sum laws, we can deduce the rest masses and intrinsic quantum numbers of baryons and meson from the deduced quarks. The deduced quantum numbers match experimental results exactly. The deduced rest masses are consistent with experimental results. This paper predicts some new quarks $[d_s(773), d_s(1933), u_c(6073), d_b(9333)]$, baryons $[\Lambda_c(6699), \Lambda_b(9959)]$ and mesons $[D(6231), B(9502)]$. PACS: 12.60.-i; 12.39.-x; 14.65.-q; 14.20.-c Key word: beyond the standard model

1. Introduction

One hundred years ago, classic physics had already been fully developed. Most physical phenomena could be explained with this physics. Black body spectrum, however, could not be explained by the physics of that time, leading Planck to propose a quantization postulate to solve this problem [1]. The Planck postulate eventually led to quantum mechanics. Physicists already clearly knew that the black body spectrum was a new phenomenon outside the applicable area of classic physics. The development from clas-

sic physics to quantum physics depended mainly on new physical ideas (a quantization postulate) rather than complex mathematics and extra dimensions of space.

Today we face a similar situation. The standard model [2] “is in excellent accord with almost all current data.... It has been enormously successful in predicting a wide range of phenomena,” but it cannot deduce the mass spectra of quarks. So far, no theory has been able to successfully do so. Like black body spectrum, this mass spectrum may need a new theory outside the standard model. M. K. Gaillard, P. D. Grannis, and F. J. Sciulli have already pointed out [2] that the standard model “is incomplete... We do not expect the standard model to be valid at arbitrarily short distances. However, its remarkable success strongly suggest that the standard model will remain an excellent approximation to nature at distance scales as small as 10^{-18}m ... high degree of arbitrariness suggests that a more fundamental theory underlies the standard model.” The mass spectrum of quarks is outside the applicable area of the standard model. Physicists need to find a new and more fundamental theory that underlies the standard model. The history of quantum physics shows that a new physics theory’s primary need is new physical ideas. A three step quantization is the new physical idea. Using this quantization, we try to deduce the masses of quarks.

Today, physics’ foundation is quantum physics (quantum mechanics and quantum field theory), not classic physics. We work with quantum systems (quarks and hadrons) that are a level deeper than the system (atoms and molecules) faced by Planck and Bohr. Therefore, the quantized systems are quantum systems (not the classic system). If Planck and Bohr got correct quantizations for atoms and molecules using only one simple quantization, we must use more steps and more complex quantizations. It is worth emphasizing that deducing the rest masses and the intrinsic quantum numbers of the quarks using the three step quantization may be one level deeper than the standard model. Hopefully, the three step quantization can help physicists discover a more fundamental theory underlying the standard model [2], just as the Planck-Bohr quantization

did.

2. The elementary quarks and their free excited quarks

2.1 The elementary quarks

We assume that there is only one elementary quark family ϵ with $s = I = \frac{1}{2}$ and two isospin states (ϵ_u has $I_Z = \frac{1}{2}$ and $Q = +\frac{2}{3}$, ϵ_d has $I_Z = -\frac{1}{2}$ and $Q = -\frac{1}{3}$). For ϵ_u (or ϵ_d), there are three colored (red, yellow and blue) quarks. Thus, there are six Fermi elementary quarks in the ϵ family with $S = C = B = 0$ in the vacuum. ϵ_u and ϵ_d have SU(2) symmetry.

As a colored (red, yellow or blue) elementary quark ϵ_u (or ϵ_d) is excited from the vacuum, its color, electric charge, rest mass and spin do not change, but it will get energy. The free excited state of the elementary quark ϵ_u is the u-quark with either a red, yellow or blue color, $Q = \frac{2}{3}$, rest mass $m_{\epsilon_u}^*$, $I = s = \frac{1}{2}$ and $I_z = \frac{1}{2}$. The free excited state of the elementary quark ϵ_d is the d-quark with either a red, yellow or blue color, $Q = -\frac{1}{3}$, rest mass $m_{\epsilon_d}^*$, $I = s = \frac{1}{2}$ and $I_z = -\frac{1}{2}$. Since ϵ_u and ϵ_d have SU(2) symmetry, the u-quark and the d-quark also have SU(2) symmetry.

2.2 The free motion of excited quark

For the excited quark free motion, we generally use the Dirac equation,

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar c}{i} (\alpha_1 \frac{\partial \psi}{\partial x^1} + \alpha_2 \frac{\partial \psi}{\partial x^2} + \alpha_3 \frac{\partial \psi}{\partial x^3}) + \beta m_{\epsilon}^* C^2 \psi. \quad (1)$$

Our purpose, however, is to find rest masses of the excited quarks. The rest masses are the energy of the excited quark at rest. Corresponding to an excited quark at rest, the

free motion Dirac equation (1) reduces [3] to

$$i\hbar \frac{\partial \psi}{\partial t} = \beta m_c^* c^2 \psi. \quad (2)$$

The equation has four solutions:

$$\psi^1 = e^{-i\frac{mc^2}{\hbar}t} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \psi^2 = e^{-i\frac{mc^2}{\hbar}t} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \psi^3 = e^{+i\frac{mc^2}{\hbar}t} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \psi^4 = e^{+i\frac{mc^2}{\hbar}t} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

ψ^1 and ψ^2 correspond to positive energy (quark) and ψ^3 and ψ^4 correspond to negative energy (antiquark). If we only consider quark and omit antiquark, we can get the two-component Pauli equation. Bjorken and Drell wrote [3]: “In particular, we wish to show that they have a sensible nonrelativistic reduction to the two-component Pauli spin theory.” Thus we can use the low energy limit of the Dirac equation– Pauli equation to find the rest masses of quarks.

For single quark free low energy motion, the quark with spin up ($s_z = \frac{1}{2}$) will have the same energy as the same quark with spin down ($s_z = -\frac{1}{2}$). Thus, for single free low energy limits, we can omit the spin of the quark, and the two-component Pauli equation can be approached by the Schrödinger [4] equation. The Schrödinger equation is not to be looked down on. In fact, it is useful in deducing the rest masses of some baryons [5] and “quarkonium” mesons $c\bar{c}$ and $b\bar{b}$ [6] and [7].

When we use the Schrödinger equation to approach the Pauli equation, we cannot forget the static energy of the excited quark. We will deal with this energy as a constant potential energy (V) at any location. The approximate Schrödinger equation is:

$$\frac{\hbar^2}{2m_c^*} \nabla^2 \psi + (E - V) \psi = 0 \quad (3)$$

where m_ϵ^* is the unknown rest mass of the excited quark. V is the static energy (constant), and it is the minimum excited energy of an elementary quark from the vacuum. The solution of (3) is the eigen wave function and the eigen energy of the free u-quark or the free d-quark:

$$\begin{aligned} \text{eigen function } \psi_{\vec{k}}(\vec{r}) &\sim \exp(i \vec{k} \cdot \vec{r}), \\ \text{eigen energy } \mathbb{E} &= V + \frac{\hbar^2}{2m_\epsilon^*}[(k_1)^2 + (k_2)^2 + (k_3)^2]. \end{aligned} \quad (4)$$

According to the Quark Model [8] a proton $p = uud$ and a neutron $n = udd$. Omitting electromagnetic mass of quarks, from (4), at $\vec{k} = 0$, we have the rest masses

$$M_p = m_u^* + m_u^* + m_d^* - |E_{bind}| \approx M_n = m_u^* + m_d^* + m_d^* - |E_{bind}| = 939 \text{ Mev} \quad (5)$$

$$\rightarrow m_u^* = m_d^* = V = \frac{1}{3}(939 + |E_{bind}|) = 313 + \Delta \text{ (Mev)} \quad (6)$$

where E_{bind} is the total binding energy of the three quarks in a baryon. Δ represents $\frac{1}{3}|E_{bind}|$, and is an unknown large positive constant. Since no free quark has been found, we assume

$$\Delta = \frac{1}{3}|E_{bind}| \gg M_p. \quad (7)$$

The free excited $u(313+\Delta)$ -quark and $d(313+\Delta)$ -quark have large rest masses $(313+\Delta) \gg M_p = 938 \text{ Mev}$. This is a reason that the Schrödinger equation of the low energy free quark is a good approximation of the Dirac equation. The large rest masses of excited quarks guarantee that the Schrödinger equation is a very good approximation.

Now we deduce the energy bands (quarks) with a three step quantization method.

3 Deducing energy bands with a three step Quantization

Today's physics must continue to develop from the standard model into a more fundamental physics [2]. The new theory will deduce the rest masses and intrinsic quantum numbers (I, S, C, B and Q) of the quarks. Recall that the development from classic physics to quantum physics began with the Planck-Bohr quantization. We try to use a three step quantization method to start the long time procedure.

3.1 Recall Planck and Bohr's works

Planck's [1] energy quantization postulate states that "any physical entity whose single 'coordinate' execute simple harmonic oscillations (i.e., is a sinusoidal function of time) can possess only total energy ε which satisfy the relation $\varepsilon = nh\nu$, $n = 0, 1, 2, 3, \dots$ where ν is the frequency of the oscillation and h is a universal constant." Planck selects reasonable energy from a continuous energy spectrum.

Bohr's [9] orbit quantization tells us that "an electron in an atom moves in a circular orbit about the nucleus...obeying the laws of classical mechanics. But instead of the infinity of orbits which would be possible in classical mechanics, it is only possible for an electron to move in an orbit for which its orbital angular momentum L is an integral multiple of Planck's constant h , divided by 2π ." Using the quantization condition ($L = \frac{nh}{2\pi}$, $n = 1, 2, 3, \dots$), Bohr selects reasonable orbits from infinite orbits.

Drawing from these great physicists' works, we find that the most important law is to use quantized conditions and symmetries (as circular orbit) to select reasonable energy levels from a continuous energy spectrum.

3.2 Three step Quantization

In order to get the short-lived quarks, we quantize the free motion (quantum plane wave function of the Schrödinger equation) of an excited quark (4) to select energy bands

from the continuous energy. The energy bands will correspond to short-lived quarks.

3.2.1 The first step quantizing condition

For free motion of an excited quark with continuous energy (4), we assume the wave vector \vec{k} has the symmetries of the regular rhombic dodecahedron in \vec{k} -space (see Fig. 1).

We assume that the axis Γ -H in Fig.1 has length $\frac{2\pi}{a}$ with an unknown constant a . The first step quantizing conditions are:

$$\begin{aligned} k_1 &= \frac{2\pi}{a}(n_1 - \xi), \\ k_2 &= \frac{2\pi}{a}(n_2 - \eta), \\ k_3 &= \frac{2\pi}{a}(n_3 - \zeta). \end{aligned} \tag{8}$$

Putting (8) into (4), we get (9) and (10):

$$\psi_{\vec{k}}(\vec{r}) \approx \exp \frac{2\pi i}{a} [(n_1 - \xi)x + (n_2 - \eta)y + (n_3 - \zeta)z], \tag{9}$$

$$\begin{aligned} E(\vec{k}, \vec{n}) &= 313 + \Delta + \alpha [(n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \zeta)^2] \\ &= 313 + \Delta + 360 E(\vec{\kappa}, \vec{n}) \text{ (Mev)}. \end{aligned} \tag{10}$$

where $\alpha = \frac{\hbar^2}{2m_e^* a^2} = 360 \text{ Mev}$ (29). $E(\vec{\kappa}, \vec{n}) = [(n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \zeta)^2]$. For $\vec{n} = (n_1, n_2, n_3)$, n_1, n_2 and n_3 are \pm integers and zero. The $\vec{\kappa} = (\xi, \eta, \zeta)$ has the symmetries of a regular rhombic dodecahedron. In order to deduce the short-lived quarks, we must further quantize the \vec{n} and $\vec{\kappa}$ as follows.

3.2.2 The second step quantization

We will quantize the $\vec{n} = (n_1, n_2, n_3)$ values further. If we assume $n_1 = l_2 + l_3$, $n_2 = l_3 + l_1$ and $n_3 = l_1 + l_2$, so that

$$\begin{aligned} l_1 &= \frac{1}{2}(-n_1 + n_2 + n_3) \\ l_2 &= \frac{1}{2}(+n_1 - n_2 + n_3) \\ l_3 &= \frac{1}{2}(+n_1 + n_2 - n_3). \end{aligned} \tag{11}$$

The second step quantizing condition is that only those values of $\vec{n} = (n_1, n_2, n_3)$ are allowed that make $\vec{l} = (l_1, l_2, l_3)$ an integer vector. This is a second step quantization for \vec{n} values. For example, \vec{n} cannot take the values (1, 0, 0) or (1, 1, -1), but can take (0, 0, 2) and (1, -1, 2). From $E(\vec{\kappa}, \vec{n}) = [(n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \varsigma)^2]$, we can give a definition of the equivalent \vec{n} : for $\vec{\kappa} = (\xi, \eta, \varsigma) = 0$, all \vec{n} values that give the same $E(\vec{\kappa}, \vec{n})$ value are equivalent n-values. We show the low level equivalent \vec{n} -values that satisfy condition (11) in the following list (12) (note $\bar{n}_i = -n_i$):

$E(\vec{n}, 0) = 0 : (0, 0, 0)$	Notes: $[\bar{1}12 \equiv (-1, 1, 2) \text{ and } \bar{1}1\bar{2} \equiv (-1, 1, -2)]$	(12)
$E(\vec{n}, 0) = 2 : (101, \bar{1}01, 011, 0\bar{1}1, 110, 1\bar{1}0, \bar{1}10, \bar{1}\bar{1}0, 10\bar{1}, \bar{1}0\bar{1}, 01\bar{1}, 0\bar{1}\bar{1})$		
$E(\vec{n}, 0) = 4 : (002, 200, 200, \bar{2}00, 0\bar{2}0, 00\bar{2})$		
$E(\vec{n}, 0) = 6 :$	$112, 211, 121, \bar{1}21, \bar{1}12, 2\bar{1}1, 1\bar{1}2, 21\bar{1}, 12\bar{1}, \bar{2}11, 1\bar{2}1, 11\bar{2},$ $\bar{1}\bar{1}2, \bar{1}2\bar{1}, 2\bar{1}\bar{1}, \bar{2}\bar{1}\bar{1}, \bar{1}21, 11\bar{2}, 12\bar{1}, \bar{1}1\bar{2}, \bar{2}1\bar{1}, \bar{2}1\bar{1}, \bar{1}2\bar{1}, \bar{1}1\bar{2},$	

3.2.3 The third step Quantization

The vector $\vec{\kappa} = (\xi, \eta, \varsigma)$ in (9) and (10) has the symmetries of the regular rhombic dodecahedron in k-space (see Fig. 1). From Fig. 1, we can see that there are four kinds of symmetry points (Γ , H, P and N) and six kinds of symmetry axes (Δ , Λ , Σ , D, F and G) in the regular rhombic dodecahedron. The coordinates (ξ, η, ς) of the symmetry

points and axes are:

$$\vec{\kappa}_\Gamma = (0, 0, 0), \vec{\kappa}_\text{H} = (0, 0, 1), \vec{\kappa}_\text{P} = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \vec{\kappa}_\text{N} = \left(\frac{1}{2}, \frac{1}{2}, 0\right). \quad (13)$$

$$\begin{aligned} \vec{\kappa}_\Delta &= (0, 0, \zeta), \quad 0 \leq \zeta \leq 1; & \vec{\kappa}_\Lambda &= (\xi, \xi, \xi), \quad 0 \leq \xi \leq \frac{1}{2}; \\ \vec{\kappa}_\Sigma &= (\xi, \xi, 0), \quad 0 \leq \xi \leq \frac{1}{2}; & \vec{\kappa}_\text{D} &= \left(\frac{1}{2}, \frac{1}{2}, \xi\right), \quad 0 \leq \xi \leq \frac{1}{2}; \\ \vec{\kappa}_\text{G} &= (\xi, 1-\xi, 0), \quad \frac{1}{2} \leq \xi \leq 1; & \vec{\kappa}_\text{F} &= (\xi, \xi, 1-\xi), \quad 0 \leq \xi \leq \frac{1}{2}. \end{aligned} \quad (14)$$

The third step quantizing condition is that the coordinates (ξ, η, ζ) of $\vec{\kappa}$ in (10) only take the coordinate values of the six symmetry axes (14).

3.3 Energy bands

From the low energy free wave motion of a excited elementary quark ϵ with a continuous energy spectrum $\{\mathbb{E} = V + \frac{\hbar^2}{2m}[(k_1)^2 + (k_2)^2 + (k_3)^2] \text{ (4)}\}$, using the three step quantization, we obtain a new energy formula $\{\mathbb{E}(\vec{k}, \vec{n}) = 313 + \Delta + \alpha[(n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \zeta)^2] \text{ (10)}\}$ with quantized \vec{n} values of (12) and \vec{k} values of (14). The energy (10) with a $\vec{n} = (n_1, n_2, n_3)$ of (12) and a $\vec{k} = (\xi, \eta, \varsigma)$ of (14) forms an energy band. The formula (10) with quantized \vec{n} of (12) and \vec{k} of (14) is the formula that can deduced all energy bands.

3.4 Deducing energy bands

After getting (10), (12) and (14), we can deduce low energy bands of the six symmetry axes. As an example, we will deduce the single energy bands of the Δ -axis. For the Δ -axis, $\vec{\kappa}_\Delta = (0, 0, \zeta)$ from (14). Putting $\vec{\kappa}_\Delta = (0, 0, \zeta)$ into (10), we get $\mathbb{E}_\Delta(\vec{k}, \vec{n}) = 313 + \Delta + 360[(n_1)^2 + (n_2)^2 + (n_3 - \zeta)^2]$. For point Γ , $\vec{\kappa}_\Gamma = (0, 0, 0)$ from (13),

$E_{\Gamma}(\vec{\kappa}, \vec{n}) = (n_1)^2 + (n_2)^2 + (n_3)^2$. For point-H, $\vec{\kappa}_H = (0, 0, 1)$ from (13), $E_H(\vec{\kappa}, \vec{n}) = (n_1)^2 + (n_2)^2 + (n_3 - 1)^2$.

$$\begin{aligned} \text{The } \Delta\text{-axis, } \mathbb{E}_{\Delta}(\vec{k}, \vec{n}) &= 313 + \Delta + 360[(n_1)^2 + (n_2)^2 + (n_3 - \zeta)^2] \\ \text{the } \Gamma\text{-point, } \vec{\kappa}_{\Gamma} &= (0, 0, 0), E_{\Gamma}(\vec{\kappa}, \vec{n}) = (n_1)^2 + (n_2)^2 + (n_3)^2 \\ \text{the H-point, } \vec{\kappa}_H &= (0, 0, 1), E_H(\vec{\kappa}, \vec{n}) = (n_1)^2 + (n_2)^2 + (n_3 - 1)^2. \end{aligned} \quad (15)$$

Putting (n_1, n_2, n_3) values of the single bands of the Δ -axis (Table B1 of [10]) into $\mathbb{E}_{\Delta}(\vec{k}, \vec{n})$, $E_{\Gamma}(\vec{\kappa}, \vec{n})$ and $E_H(\vec{\kappa}, \vec{n})$ (15), we can find energy bands as shown in Table 1:

Table 1 The Single Energy Bands of the Δ -Axis (the Γ -H axis)				
(n_1, n_2, n_3)	$E(\vec{\kappa}, \vec{n})_{Start}$	minim. E	$\mathbb{E}(\vec{k}, \vec{n})$ (Band)	$E(\vec{\kappa}, \vec{n})_{end}$
(0, 0, 0)	$E_{\Gamma}=0$	$313+\Delta$	$313+\Delta+\zeta^2$	$E_H=1$
(0, 0, 2)	$E_H=1$	$673+\Delta$	$313+\Delta+(2-\zeta)^2$	$E_{\Gamma}=4$
(0, 0, $\bar{2}$)	$E_{\Gamma}=4$	$1753+\Delta$	$313+\Delta+(2+\zeta)^2$	$E_H=9$
(0, 0, 4)	$E_H=9$	$3553+\Delta$	$313+\Delta+(4-\zeta)^2$	$E_{\Gamma}=16$
(0, 0, $\bar{4}$)	$E_{\Gamma}=16$	$6073+\Delta$	$313+\Delta+(4+\zeta)^2$	$E_H=25$
(0, 0, 6)	$E_H=25$	$9313+\Delta$	$313+\Delta+(6-\zeta)^2$	$E_{\Gamma}=36$

$E(\vec{\kappa}, \vec{n})_{Start}$ is the value of $E(\vec{\kappa}, \vec{n})$ at the start point of the energy band				
$E(\vec{\kappa}, \vec{n})_{end}$ is the value of $E(\vec{\kappa}, \vec{n})$ at the end point of the energy band				

Similarly, we can deduce the single energy bands of the Σ -axis. For the Σ -axis, $\vec{\kappa}_{\Sigma} = (\xi, \xi, 0)$. Putting the $\vec{\kappa}$ into (10), we have $\mathbb{E}_{\Sigma}(\vec{k}, \vec{n}) = 313 + \Delta + 360[(n_1 - \xi)^2 + (n_2 - \xi)^2 + (n_3)^2]$ $0 \leq \xi \leq \frac{1}{2}$. For point N, $\vec{\kappa}_N = (\frac{1}{2}, \frac{1}{2}, 0)$ from (13), $E_N(\vec{\kappa}, \vec{n}) = (n_1 - \frac{1}{2})^2 + (n_2 - \frac{1}{2})^2 + (n_3)^2$.

$$\begin{aligned} \mathbb{E}_{\Sigma}(\vec{k}, \vec{n}) &= 313 + \Delta + 360[(n_1 - \xi)^2 + (n_2 - \xi)^2 + (n_3)^2] \\ \vec{\kappa}_N &= (\frac{1}{2}, \frac{1}{2}, 0), E_N(\vec{\kappa}, \vec{n}) = (n_1 - \frac{1}{2})^2 + (n_2 - \frac{1}{2})^2 + (n_3)^2 \\ \vec{\kappa}_{\Gamma} &= (0, 0, 0), E_{\Gamma}(\vec{\kappa}, \vec{n}) = (n_1)^2 + (n_2)^2 + (n_3)^2 \end{aligned} \quad (16)$$

Putting (n_1, n_2, n_3) values of the single bands of the Σ -axis (Table B2 of [10]) into $\mathbb{E}(\vec{k}, \vec{n})$, $E_N(\vec{\kappa}, \vec{n})$ and $E_{\Gamma}(\vec{\kappa}, \vec{n})$ (16), we can deduce energy bands as shown in Table 2

Table 2 The Single Energy Bands of the Σ -Axis (the Γ -N axis)				
$n_1 n_2 n_3$	$E(\vec{\kappa}, \vec{n})_{Start}$	minim.E	$\mathbb{E}_{\Sigma}=313++\Delta+360E(\vec{\kappa}, \vec{n})$	$E(\vec{\kappa}, \vec{n})_{end}$
(0, 0, 0)	$E_{\Gamma}=0$	$313+\Delta$	$313+\Delta+720\xi^2$	$E_N=\frac{1}{2}$
(1,1,0)	$E_N=\frac{1}{2}$	$493+\Delta$	$313+\Delta+720(1-\xi)^2$	$E_{\Gamma}=2$
(-1,-1,0)	$E_{\Gamma}=2$	$1033+\Delta$	$313+\Delta+720(1+\xi)^2$	$E_N=\frac{9}{2}$
(2,2,0)	$E_N=\frac{9}{2}$	$1933+\Delta$	$313+\Delta+720(2-\xi)^2$	$E_{\Gamma}=8$
(-2,-2,0)	$E_{\Gamma}=8$	$3193+\Delta$	$313+\Delta+720(2+\xi)^2$	$E_N=\frac{25}{2}$
(3,3,0)	$E_N=\frac{25}{2}$	$4813+\Delta$	$313+\Delta+720(3-\xi)^2$	$E_{\Gamma}=18$
(-3,-3,0)	$E_{\Gamma}=18$	$6793+\Delta$	$313+\Delta+720(3+\xi)^2$	$E_N=\frac{49}{2}$
(4, 4, 0)	$E_N=\frac{49}{2}$	$9133+\Delta$	$313+\Delta+720(4-\xi)^2$	$E_{\Gamma}=32$
$E(\vec{\kappa}, \vec{n})_{Start}$ is the value of $E(\vec{\kappa}, \vec{n})$ at the start point of the energy band				
$E(\vec{\kappa}, \vec{n})_{end}$ is the value of $E(\vec{\kappa}, \vec{n})$ at the end point of the energy band				

Following the two examples in Tables 1 and 2, we can deduce all low energy bands of the six axes using the formulae (10), (12) and (14) (see Appendix B of [10])

From these energy bands, we can deduce quarks using phenomenological formulae.

4 The phenomenological formulae

In order to deduce the short-lived quarks from energy bands, we assume the following phenomenological formulae for the rest mass and intrinsic quantum numbers (I, S, C, B and Q) of energy bands:

1). For a group of degenerate energy bands (number = deg) with the same energy and equivalent \vec{n} values (12), the isospin is

$$2I + 1 = \text{deg} \rightarrow I = \frac{\text{deg} - 1}{2} \quad (17)$$

2). The strange number S of an energy band (quark) that lies on an axis with a rotary fold R of the regular rhombic dodecahedron is

$$S = R - 4. \quad (18)$$

3). For energy bands with $\deg < R$ and $R - \deg \neq 2$ (such as the single energy bands on the Γ -H axis and the Γ -N axis), the strange number is

$$S = S_{axis} + \Delta S, \quad \Delta S = \delta(\tilde{n}) + [1 - 2\delta(S_{axis})]\text{Sign}(\tilde{n}) \quad (19)$$

where $\delta(\tilde{n})$ and $\delta(S_{axis})$ are Dirac functions and S_{axis} is the strange number (18) of the axis. For an energy band with $\vec{n} = (n_1, n_2, n_3)$, \tilde{n} is defined as

$$\tilde{n} \equiv \frac{n_1 + n_2 + n_3}{|n_1| + |n_2| + |n_3|}, \quad \text{Sgn}(\tilde{n}) = \begin{bmatrix} +1 & \text{for } \tilde{n} > 0 \\ 0 & \text{for } \tilde{n} = 0 \\ -1 & \text{for } \tilde{n} < 0 \end{bmatrix} \quad (20)$$

$$\text{If } \tilde{n} = 0 \quad \Delta S = \delta(0) = +1 \quad \text{from (19) and (20).} \quad (21)$$

$$\text{If } \tilde{n} = \frac{0}{0}, \text{ we assume } \Delta S = -S_{axis}. \quad (22)$$

Thus, for $\vec{n} = (0, 0, 0)$, from (22), we have

$$S = S_{axis} + \Delta S = S_{axis} - S_{axis} = 0. \quad (23)$$

4). If an energy band with $S = +1$, we call it has a charmed number C ($C = 1$):

$$\text{if } \Delta S = +1 \rightarrow S = S_{Ax} + \Delta S = +1, \quad C \equiv +1. \quad (24)$$

If an energy band with $S = -1$, which originates from $\Delta S = +1$ ($S_{Ax} = -2$), and there is an energy fluctuation, we call it has a bottom number B :

$$\text{for single bands, if } \Delta S = +1 \rightarrow S = -1 \text{ and } \Delta E \neq 0, \quad B \equiv -1. \quad (25)$$

5). The elementary quark ϵ_u (or ϵ_d) determines the electric charge Q of an excited quark. For an excited quark of ϵ_u (or ϵ_d), $Q = +\frac{2}{3}$ (or $-\frac{1}{3}$). For an excited quark with

isospin I, there are $2I + 1$ members . For $I_z > 0$, $Q = +\frac{2}{3}$; $I_z < 0$, $Q = -\frac{1}{3}$ and $I_z = 0$,

$$\text{if } S+C+b > 0, Q = Q \quad \epsilon_u(0) = \frac{2}{3}; \quad (26)$$

$$\text{if } S+C+b < 0, Q = Q \quad \epsilon_d(0) = -\frac{1}{3}. \quad (27)$$

There is no quark with $I_z = 0$ and $S + C + b = 0$.

6). Since the experimental full width of baryons is about 100 Mev order, for simplicity, we assume that a fluctuation energy ΔE of a quark roughly is

$$\Delta E = 100 S[(1+S_{Ax})(J_S+S_{Ax})]\Delta S \quad J_S = |S_{Ax}| + 1, 2, 3, \dots \quad (28)$$

Fitting experimental results, we can get

$$\alpha = 360 \text{ Mev}. \quad (29)$$

The rest mass (m^*) of a quark is the minimum energy of the energy band. From (10), (29) and (28), the rest mass (m^*) of the quark is

$$\begin{aligned} m^* &= \{313 + 360 \text{ minimum}[(n_1-\xi)^2 + (n_2-\eta)^2 + (n_3-\zeta)^2] + \Delta E + \Delta\} \text{ (Mev)} \\ &= m + \Delta \text{ (Mev)}, \end{aligned} \quad (30)$$

This formula (30) is the united quark mass formula.

Using above phenomenological formulae, we will show that each energy band corresponds to a short-lived quark and deduce its intrinsic quantum numbers I, S, C, B and Q. The minimum energy of the energy band is the rest mass of the short-lived quark corresponding to the energy band.

5 Deducing quarks using the phenomenological formulae from energy bands

5.1 Deducing quarks from the energy bands in Tables 1 and 2

From deduced single energy bands of the Δ -axis in Table 1, we can use the above formulae (17)-(30) to deduce the quarks. For the Δ -axis, $R = 4$, strange number $S_\Delta = 0$ from (18). For single energy bands, $I = 0$ from (17); and $S = S_\Delta + \Delta S = \Delta S = \delta(\tilde{n}) - \text{Sign}(\tilde{n})$ from (19). For $\vec{n} = (0, 0, -2)$ and $(0, 0, -4)$, $\Delta S = +1$ from (20) and (19); for $n = (0, 0, 2)$, $(0, 0, 4)$ and $(0, 0, 6)$ $\Delta S = -1$ from (20) and (19). Using (24), (20) and (19), we can find the charmed number $C = +1$ when $n = (0, 0, -2)$ and $(0, 0, -4)$. From (26), we can find $Q = \frac{2}{3}$ when $n = (0, 0, -2)$ and $(0, 0, -4)$; from (27), $Q = -\frac{1}{3}$ when $n = (0, 0, 2)$, $(0, 0, 4)$ and $(0, 0, 6)$. From (28) and (30), we can find the rest masses $(\text{minimE} + \Delta E)$. We list all results in Table 3:

Table 3. The $u_C(m^*)$ -quarks and the $d_S(m^*)$ -quarks on the Δ -axis											
$S_{axis} = 0, I = 0, S = \Delta S = \delta(\tilde{n}) + [1-2\delta(S_{axis})]\text{Sign}(\tilde{n}), \tilde{n} \equiv \frac{n_1+n_2+n_3}{ n_1 + n_2 + n_3 }$											
n_1, n_2, n_3	E_{Point}	MinimE	ΔS	J	I	S	C	Q	ΔE	$q_{Name}(m^*)$	
0, 0, 0	$E_\Gamma=0$	313	0	$J = 0$	$\frac{1}{2}$	0	0	$\frac{2}{3}$	0	$u(313+\Delta)$	
0, 0, 2	$E_H=1$	673	-1	$J_{S,H}=1$	0	-1	0	$-\frac{1}{3}$	100	$d_S(773+\Delta)$	
0, 0, -2	$E_\Gamma=4$	1753	+1	$J_{C,\Gamma}=1$	0	0	1	$\frac{2}{3}$	0	$u_C(1753+\Delta)$	
0, 0, 4	$E_H=9$	3553	-1	$J_{S,H}=2$	0	-1	0	$-\frac{1}{3}$	200	$d_S(3753+\Delta)$	
0, 0, -4	$E_\Gamma=16$	6073	+1	$J_{C,\Gamma}=2$	0	0	1	$\frac{2}{3}$	0	$u_C(6073+\Delta)$	
0, 0, 6	$E_H=25$	9313	-1	$J_{S,H}=3$	0	-1	0	$-\frac{1}{3}$	300	$d_S(9613+\Delta)$	

Similarly, for the Σ -axis, $R = 2$, strange number $S_{axis} = -2$ from (18). For single energy bands, $I = 0$ from (17). From (19), $S = S_{axis} + \Delta S = -2 + \Delta S$; the $\Delta S = \delta(\tilde{n}) + \text{Sign}(\tilde{n})$. For $\vec{n} = (1, 1, 0)$, $(2, 2, 0)$, $(3, 3, 0)$ and $(4, 4, 0)$, $\Delta S = +1$ from (20) and (19); for $\vec{n} = (-1, -1, 0)$, $(-2, -2, 0)$ and $(-3, -3, 0)$, $\Delta S = -1$ from (20) and (19). Using (25), (20), (19) and (28), we can find the bottom number $B = -1$ when $\vec{n} = (3, 3, 0)$ and $(4, 4, 0)$. From (26) and (27), we can find the electric charge $Q = -\frac{1}{3}$ for all quarks. From (28) and (30), we can deduce rest masses $(\text{minimE} + \Delta E)$ of quarks from the energy bands in Table 2. We list all results in Table 4:

Table 4. The $d_b(m^*)$ -Quarks, $d_s(m^*)$ -Quarks and $d_\Omega(m^*)$ -Quarks of the Σ -Axis										
$S_{axis} = -2, I = 0, S = S_{axis} + \Delta S = \delta(\tilde{n}) + [1-2\delta(S_{axis})]\text{Sign}(\tilde{n}), \tilde{n} \equiv \frac{n_1+n_2+n_3}{ n_1 + n_2 + n_3 }$										
E_{Point}	$n_1 n_2 n_3$	ΔS	S	B	Q	J	I	minimE	ΔE	$q_{Name}(m^*)$
$E_\Gamma=0$	(0, 0, 0)	+2*	0	0	$\frac{-1}{3}$	$J_{S,\Gamma}=0$	$\frac{1}{2}$	313	0	$d(313+\Delta)$
$E_N=\frac{1}{2}$	(1, 1, 0)	+1	-1	0	$\frac{-1}{3}$	$J_{S,N}=1$	0	493	0	$d_s(493+\Delta)$
$E_\Gamma=2$	(-1,-1,0)	- 1	-3	0	$\frac{-1}{3}$	$J_{S,\Gamma}=1$	0	1033	0	$d_\Omega(1033+\Delta)$
$E_N=\frac{9}{2}$	(2, 2, 0)	+1	-1	0	$\frac{-1}{3}$	$J_{S,N}=2$	0	1933	0	$d_s(1933+\Delta)$
$E_\Gamma=8$	(-2,-2,0)	- 1	-3	0	$\frac{-1}{3}$	$J_{S,\Gamma}=2$	0	3193	0	$d_\Omega(3193+\Delta)$
$E_N=\frac{25}{2}$	(3, 3, 0)	+1	0	-1	$\frac{-1}{3}$	$J_{S,N}=3$	0	4813	100	$d_B(4913+\Delta)$
$E_\Gamma=18$	(-3,-3,0)	- 1	-3	0	$\frac{-1}{3}$	$J_{S,\Gamma}=3$	0	6793	-300	$d_\Omega(6493+\Delta)$
$E_N=\frac{49}{2}$	(4, 4, 0)	+1	0	-1	$\frac{-1}{3}$	$J_{S,N}=4$	0	9133	200	$d_B(9333+\Delta)$
*For $\vec{n} = (n_1, n_2, n_3) = (0, 0, 0)$, $\Delta S = - S_{axis} = +2$ from (22)										

5.2 The five deduced Ground quarks

From Table 3 and Table 4, we find: The unflavored ($S = C = B = 0$) ground quarks are $u(313+\Delta)$ and $d(313+\Delta)$. The strange quarks $d_s(493)$, $d_s(773)$, $d_s(1933)$, $d_s(3753)$, $d_s(9613)$, $d_\Omega(1033+\Delta)$, $d_\Omega(3193+\Delta)$ and $d_\Omega(6493+\Delta)$; The ground strange quark is $d_s(493)$. The charmed quarks $u_c(1753)$ and $u_c(6073)$; the charmed ground quark is $u_c(1753)$. The bottom quarks $d_b(4913)$ and $d_b(9333)$; the bottom ground quark is $d_b(4913)$. (in Table 11 of [10] we have shown all low energy quarks, the five deduced ground quarks are still the ground quarks of all quarks). For the four flavored quarks, there are five ground quarks [$u(313+\Delta)$, $d(313+\Delta)$, $d_s(493+\Delta)$, $u_c(1753+\Delta)$ and $d_b(4913+\Delta)$] in the quark spectrum. These five ground quarks correspond to the five quarks [11] of the current Quark Model: $u \leftrightarrow u(313+\Delta)$, $d \leftrightarrow d(313+\Delta)$, $s \leftrightarrow d_s(493+\Delta)$, $c \leftrightarrow u_c(1753+\Delta)$ and $b \leftrightarrow d_b(4913+\Delta)$.

We can compare the rest masses and intrinsic quantum numbers (I, S, C, b and Q) [11] of the current quarks with the deduced values of the five ground quarks. The deduced intrinsic quantum numbers (I, S, C, b and Q) of the five ground quarks are

exactly the same as the five current quarks as shown in Table 5A:

Table 5A. The Five Deduced Ground Quarks and Current Quarks

Dq(m) [§] , Cq*	u(313), u		d(313), d		d _s (493), s		u _c (1753), c		d _b (4913), b	
Strange S	0	0	0	0	-1	-1	0	0	0	0
Charmed C	0	0	0	0	0	0	1	1	0	0
Bottom B	0	0	0	0	0	0	0	0	-1	-1
Isospin I	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0	0
I _Z	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	0	0
Electric Q _q	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$

Dq(m)[§]= Deduced ground quark; Cq*= Current quark

The deduced rest masses of the five ground quarks are roughly a constant (about 390 Mev) larger than the masses of the current quarks, as shown in Table 5B.

Table 5B Comparing the Rest Masses of Deduced and Current Quarks

Current Quark	Up	Down	Strange	Charmed	bottom
Current Quark(m)	u(2.8)	d(6)	s(105)	c(1225)	b(4500)
Current quark mass	1.5 to 4	4 to 8	80 to 130	1250 to 1350	4.1 to 4.4 G. 4.6 to 4.9 G.
Deduced Quark (m)	u(313)	d(313)	d _s (493)	u(1753)	d _b (4913)
m _{Curr.} -m _{Deduced.}	310	307	388	528	413

The rest mass of a deduced quark m* = m + Δ → m = m* - Δ

These mass differences may originate from different energy reference systems. If we use the same energy reference system, the deduced masses of ground quarks will be roughly consistent with the masses of the corresponding current quarks. Of course, the ultimate test is whether or not the baryons and mesons composed of the deduced quarks are consistent with experimental results.

We will deduce the baryons and mesons composed of the quarks in Table 3 and 4 in this paper.

6 Deducing the baryons of the quarks in Table 3 and 4

According to the Quark Model [8], a colorless baryon is composed of three different colored quarks. Using sum laws (31) and the deduced quarks in Tables 3 and 4, we can deduce the baryons as shown in Table 6. From Tables 3 and 4, we can see that there is a term Δ of the rest masses of quarks. Δ is a very large unknown constant. Since the rest masses of the quarks in a baryon are large (from Δ) and the rest mass of the baryon composed by three quarks is not, we infer that there will be a strong binding energy ($E_{Bind} = -3\Delta$) to cancel 3Δ from the three quarks: $M_B = m_{q_1}^* + m_{q_{N(313)}}^* + m_{q_{N(313)}}^* - |E_{Bind}| = m_{q_1} + m_{q_{N(313)}} + m_{q_{N(313)}} + 3\Delta - 3\Delta = m_{q_1} + m_{q_{N(313)}} + m_{q_{N(313)}}$. Thus we will omit the term 3Δ in the three quark masses and the term -3Δ in the binding energy from now on. For simplicity's sake, we only deduced baryons composed of at least two free excited quark $q_{N(313)}$ [u(313), d(313)] since other baryons have much lower productivity. For these baryons, sum laws are:

$$\begin{aligned}
 \text{Baryon strange number } S_B &= S_{q_1} + S_{q_{N(313)}} + S_{q_{N(313)}} = S_{q_1}, \\
 \text{baryon charmed number } C_B &= C_{q_1} + C_{q_{N(313)}} + C_{q_{N(313)}} = C_{q_1}, \\
 \text{baryon bottom number } B_B &= B_{q_1} + B_{q_{N(313)}} + B_{q_{N(313)}} = B_{q_1}, \\
 \text{baryon electric charge } Q_B &= Q_{q_1} + Q_{q_{N(313)}} + Q_{q_{N(313)}}. \\
 \text{baryon mass } M_B &= m_{q_1} + m_{q_{N(313)}} + m_{q_{N(313)}} \text{ (except charmed baryons)}
 \end{aligned} \tag{31}$$

For charmed baryons, $M_B = m_{q_1} + m_{q_{N(313)}} + m_{q_{N(313)}} + \Delta e$

$$\Delta e = 100C(2I-1) \tag{32}$$

where C is the charmed number and I is the isospin of the charmed baryons. Using sum laws (31) and binding energy formula (32), we deduce the baryons shown in Table 6 from the quarks in Tables 3 and 4:

Table 6. The Baryons of the Quarks in Table 3 and Table 4

$q_i^{Iz}(m)$	q_j	q_k	I	S	C	b	Q	M	Baryon	Exper.	$\frac{\Delta M}{M}\%$
$u^{\frac{1}{2}}(313)$	u	d	$\frac{1}{2}$	0	0	0	1	939	p(939)	p(938)	0.11
$d^{\frac{-1}{2}}(313)$	u	d	$\frac{1}{2}$	0	0	0	0	939	n(939)	n(940)	0.11
$d_s^0(493)$	u	d	0	-1	0	0	0	1119	$\Lambda(1119)$	$\Lambda^0(1116)$	0.27
$u_c^0(1753)$	u	d	0	0	1	0	1	2279	$\Lambda_c(2279)$	$\Lambda_c^+(2285)$	0.3.
$u_c^0(1753)$	u	u	1	0	1	0	2	2479	$\Sigma_c^{++}(2479)$	$\Sigma_c^{++}(2455)$	1.0
$u_c^0(1753)$	u	d	1	0	1	0	1	2479	$\Sigma_c^+(2479)$	$\Sigma_c^+(2455)$	1.0
$u_c^0(1753)$	d	d	1	0	1	0	0	2479	$\Sigma_c^-(2479)$	$\Sigma_c^-(2455)$	1.0
$d_b^0(4913)$	u	d	0	0	0	-1	0	5539	$\Lambda_b(5539)$	$\Lambda_b^0(5624)$	1.5
$d_s^0(773)$	u	d	0	-1	0	0	0	1399	$\Lambda(1399)$	$\Lambda(1405)$	0.4
$d_s^0(1933)$	u	d	0	-1	0	0	0	2559	$\Lambda(2559)$	$\Lambda(2585)^{**}$	1.0
$d_s^0(3753)$	u	d	0	-1	0	0	0	4375	$\Lambda(4375)$	Prediction	
$d_s^0(9613)$	u	d	0	-1	0	0	0	10239	$\Lambda(10239)$	Prediction	
$u_c^0(6073)$	u	d	0	0	1	0	1	6699	$\Lambda_c^+(6699)$	Prediction	
$d_b^0(9333)$	u	d	0	0	0	-1		9959	$\Lambda_b^0(9959)$	Prediction	
$d_\Omega^0(1033)$	d	d	0	-3	0	0	-1	1659	$\Omega^-(1659)$	$\Omega^-(1672)$	0.8

$u \equiv u(313)$ and $d \equiv d(313)$. $\Lambda(2585)^{**}$ Evidence of existence only fair.

Table 6 shows that the deduced intrinsic quantum numbers (I, S, C, b and Q) of the baryons match experimental results [12] exactly and that the deduced rest masses of the baryons are consistent with more than 98.5% of experimental results [12]. These are strong supports for the deduced the rest masses and intrinsic quantum numbers of the quarks.

7 Deducing the mesons of the quarks in Tables 3 and 4

According to the Quark Model [8], a colorless meson is composed of a quark q_i with a color and an antiquark \bar{q}_j with the anticolor of the quark q_i . For each flavor, the three different colored quarks have the same I, S, C, B, Q and m. Thus we can omit the color when we deduce the rest masses and intrinsic quantum numbers of the mesons. For

mesons, the sum laws are

$$\begin{aligned}
&\text{Meson strange number } S_M = S_{q_i} + S_{\bar{q}_j}, \\
&\text{meson charmed numbers } C_M = C_{q_i} + C_{\bar{q}_j}, \\
&\text{meson bottom number } B_M = B_{q_i} + B_{\bar{q}_j}, \\
&\text{meson electric charge } Q_M = Q_{q_i} + Q_{\bar{q}_j}.
\end{aligned} \tag{33}$$

There is a strong interaction between the quark and antiquark (colors), but we do not know how large it is. Since the rest masses of the quarks in mesons are large (from Δ) and the rest mass of the meson composed of the quark and antiquark is not, we infer that there will be a large portion of binding energy (-2Δ) to cancel 2Δ from the quark and antiquark and a small amount of binding energy as shown in the following

$$E_B(q_i \bar{q}_j) = -2\Delta - 337 + 100 \left[\frac{\Delta m}{m_g} + DS - \tilde{m} + \gamma(i,j) - 2I_i I_j \right] \tag{34}$$

where $\Delta = \frac{1}{3} |E_{bind}|$ (7) is $\frac{1}{3}$ of the binding energy of a baryon (an unknown large constant, $\Delta \gg m_p = 938$ Mev), $\Delta m = |m_i - m_j|$, $DS = \left| (\Delta S)_i - (\Delta S)_j \right|$. $m_g = 939$ (Mev) unless

m_i (or m_j) equals	$m_C \geq 6073$	$m_b \geq 9333$	$m_s \geq 9613$
m_g will equal to	1753 (Table 4)	4913 (Table 7)	3753 (Table 4).

(35)

$$\tilde{m} = \frac{m_i \times m_j}{m_{g_i} \times m_{g_j}} \quad m_{g_i} = m_{g_j} = 939 \text{ (Mev) unless}$$

m_i (or m_j)	$m_{q_N} = 313$	$m_{d_s} = 493$	$m_{u_c} \succeq 1753$	$m_{d_s} > 3753$,	$m_{d_b} \succeq 4913$
m_{g_j} (or m_{g_i})	313	493	1753	3753,	4913.

(36)

If q_i and q_j are both ground quarks in Table 5, $\gamma(i, j) = 0$. If q_i and q_j are not both ground quarks, for $q_i = q_j$, $\gamma(i, j) = -1$; for $q_i \neq q_j$, $\gamma(i, j) = +1$. S_i (or S_j) is the strange number of the quark q_i (or q_j). I_i (or I_j) is the isospin of the quark q_i (or q_j).

From the quarks in Tables 3 and 4, we can use (33) and (34) to deduce the rest masses and the intrinsic quantum numbers (I, S, C, b and Q) of mesons as shown in Table 7.

Table 7. The Deduced Mesons of the Quarks in Tables 3 and 4

$q_i^{\Delta S}(m_i) \overline{q_j^{\Delta S}(m_j)}$	$\frac{100\Delta m}{939}$	DS	$-100\tilde{m}$	E_{bind}	Deduced	Experiment	R
$q_N^0(313)\overline{q_N^0(313)}$	0	0	-100	- 487 [#]	$\pi(139)$	$\pi(138)$	0.7
$q_N^0(313)\overline{d_S^1(493)}$	19	1	-100	- 318	K(488)	K(494)	0.2
$d_S^1(493)\overline{d_S^1(493)}$	0	0	-100	- 437	$\eta(549)$	$\eta(548)$	0.2
$u_C^1(1753)\overline{q_N^0(313)}$	153	1	-100	- 184	D(1882)	D(1869)	0.7
$u_C^1(1753)\overline{q_S^1(493)}$	134	0	-100	- 303	$D_S(1943)$	$D_S(1969)$	0.4
$u_C^1(1753)\overline{u_C^1(1753)}$	0	0	-100	-437	$J/\psi(3069)$	$J/\psi(3097)$	0.9
$q_N^0(313)\overline{d_b^1(4913)}$	490	1	-100	153 [*]	B(5379)	B(5279)	1.9
$d_S^1(493)\overline{d_b^1(4913)}$	471	0	-100	34 [*]	$B_S(5440)$	$B_S(5370)$	1.3
$u_C^1(1753)\overline{d_b^1(4913)}$	337	0	-100	-100	$B_C(6566)$	$B_C(6400)$	2.6
$d_b^1(4913)\overline{d_b^1(4913)}$	0	0	-100	- 437	$\Upsilon(9389)$	$\Upsilon(9460)$	0.8
$d_S^{-1}(773)\overline{d_S^{-1}(773)}$	0	0	-68	-505	$\eta(1041)$	$\phi(1020)$	2.0
$d_S^{-1}(3753)\overline{d_S^{-1}(3753)}$	0	0	-1597	-2034	$\eta(5472)$	prediction	
$d_S^1(1933)\overline{d_S^1(1933)}$	0	0	-424	-861	$\eta(3005)$	$\eta_c(2980)$	0.8
$d_S^1(9333)\overline{d_S^1(9333)}$	0	0	-361	-798	$\Upsilon(17868)$	prediction	?
$u_C^1(6073)\overline{u_C^1(6073)}$	0	0	-1200	-1637	$\psi(10509)$	$\Upsilon(10355)$	1.5
$d_S^{-1}(9613)\overline{d_S^{-1}(9613)}$	0	0	-656	-1093	$\eta(18133)$	prediction	
$d_\Omega^{-1}(1033)\overline{d_\Omega^{-1}(1033)}$	0	0	-121	-558	$\eta(1508)$	$f_0(1507)$	0.7
$q_N^0(313)\overline{d_S^{-1}(773)}$	49	1	-82	-170	K(916)	K(892)	2.7
$q_N^0(313)\overline{d_S^1(1933)}$	171	1	-206	-170	K(2076)	$K_4^*(2045)$	1.5
$q_N^0(313)\overline{d_S^{-1}(3753)}$	347	1	-400	-190	K(3876)	prediction	?
$q_N^0(313)\overline{d_S^{-1}(9613)}$	248	1	-256	-145	K(9781)	prediction	?
$q_N^0(313)\overline{d_b^1(9333)}$	183	1	-190	-144	B(9502)	prediction	?
$u_C^1(6073)\overline{q_N^0(313)}$	328	1	-346.4	-155	D(6231)	prediction	?
$u_C^1(6073)\overline{d_S^1(493)}$	318.3	0	-346.4	-265	$D_s(6301)$	prediction	?
$d_S^1(493)\overline{d_S^{-1}(773)}$	30	2	-256.1	-90	$\eta(1177)$	$\eta(1170)$	0.6
$d_S^1(493)\overline{d_S^{-1}(3753)}$	347	2	-339.7	-90	$\eta(4156)$	$\psi(4159)$.07
$d_S^1(493)\overline{d_S^{-1}(9613)}$	243	2	-256.1	-50	$\eta(10056)$	$\Upsilon(10023)$	0.4
$u_C^1(1753)\overline{d_S^{-1}(773)}$	104	2	-82.3	-15	$D_S(2511)$	$D_{S_1}(2535)$	1.0
$d_S^1(9613)\overline{d_S^{-1}(773)}$	235	0	-211	-212	$\eta(10174)$	$\chi(10232)$	0.6

[#]For $q_N^0(313)\overline{q_N^0(313)}$, $I_i I_j = \frac{1}{4} \rightarrow 100(-2I_i I_j) = -50$

^{*}The total binding energy $(153-2\Delta)$ and $(34-2\Delta)$ are negative from (34)

Table 7 shows that the deduced intrinsic quantum numbers match experimental results [13] exactly. The deduced rest masses are consistent with experimental results.

8 Predictions

This paper predicts some quarks, baryons and mesons shown in the following list:

$q_i(m)$	q_j	q_k	Baryon	*	$q_i(m)\overline{q_j(m)}$	Meson	*	$q_i(m)\overline{q_i(m)}$ [§]
$u_C(6073)$	u	d	$\Lambda_C(6699)$	*	$u_C^1(6073)\overline{q_N^0(313)}$	D(6231)	*	$\psi(10509)^{\#}$
$d_b^1(9333)$	u	d	B(9959)	*	$q_N^0(313)\overline{d_b^1(9333)}$	B(9502)	*	$\Upsilon(17868)$
$d_S(773)$	u	d	$\Lambda(1399)^{\#}$	*	$q_N^0(313)\overline{d_S^{-1}(773)}$	K(916) [#]	*	$\phi(1041)^{\#}$
$d_S(1933)$	u	d	$\Lambda(2559)^{\#}$	*	$q_N^0(313)\overline{d_S^1(1933)}$	K(2076)	*	$\eta(3005)^{\#}$
$d_S(3753)$	u	d	$\Lambda(4379)$	*	$q_N^0(313)\overline{d_S^{-1}(3753)}$	K(3876)	*	$\eta(5472)$
$d_S(9613)$	u	d	$\Lambda(10239)$	*	$q_N^0(313)\overline{d_S^{-1}(9613)}$	K(9781)	*	$\eta(18133)$

[§]The last column shows the mesons of the pair quarks $[q_i(m)\overline{q_i(m)}]$ that the

$q_i(m)$ is in the first column.

$\Lambda^0(1399)^{\#}$ [experimental $\Lambda^0(1406)$ with $(\frac{\Delta M}{M}\%) = 0.5\%$],

$\Lambda(2559)^{\#}$ [experimental $\Lambda(2585)^{**}$ with $(\frac{\Delta M}{M}\%) = 1.0\%$],

K(916)[#] [experimental $K^*(892)$ with $(\frac{\Delta M}{M}\%) = 2.7\%$],

$\eta(1041)^{\#}$ [experimental $\phi(1020)$ with $(\frac{\Delta M}{M}\%) = 2\%$],

$\eta(3005)^{\#}$ [experimental $\eta_c(2980)$ with $(\frac{\Delta M}{M}\%) = 0.8\%$].

It is very important to pay attention to the $\Upsilon(3S)$ -meson (mass $m = 10,355.2 \pm 0.4$ Mev, full width $\Gamma = 26.3 \pm 3.5$ kev). We compare the mesons $J/\psi(3097)$, $\Upsilon(9460)$ and $\Upsilon(10355)$ shown as follow list

$$u_C^1(1753)\overline{u_C^1(1753)} = J/\psi(3069) \quad [J/\psi(3096.916 \pm 0.011), \Gamma = 91.0 \pm 3.2 \text{ kev}]$$

$$d_b^1(4913)\overline{d_b^1(4913)} = \Upsilon(9389) \quad [\Upsilon(9460.30 \pm 0.26), \Gamma = 53.0 \pm 1.5 \text{ kev}]$$

$$u_C^1(6073)\overline{u_C^1(6073)} = \psi(10509) \quad [\Upsilon(10.355.2 \pm 0.4), \Gamma = 26.3 \pm 3.5 \text{ kev}]$$

$\Upsilon(3S)$ has more than three times larger of a mass than $J/\psi(1S)$ ($m = 3096.916 \pm 0.011$ Mev) and more than three times longer of a lifetime than $J/\psi(1S)$ (full width Γ

$= 91.0 \pm 3.2$ keV). It is well known that the discovery of $J/\psi(1S)$ is also the discovery of charmed quark c ($u_c(1753)$) and that the discovery of $\Upsilon(9460)$ is also the discovery of bottom quark b ($d_b(4913)$). Similarly the discovery of $\Upsilon(3S)$ will be the discovery of a very important new quark—the $u_C(6073)$ -quark.

9 Discussion

1). From the low energy free wave motion of a excited elementary quark ϵ with a continuous energy spectrum $\{\mathbb{E} = V + \frac{\hbar^2}{2m}[(k_1)^2 + (k_2)^2 + (k_3)^2]\}$, using the three step quantization, we obtain a new energy formula $\{\mathbb{E}(\vec{k}, \vec{n}) = 313 + \Delta + \alpha[(n_1 - \xi)^2 + (n_2 - \eta)^2 + (n_3 - \varsigma)^2]\}$ with quantized \vec{n} values (12) and \vec{k} values (14). The energy (10) with a $\vec{n} = (n_1, n_2, n_3)$ of (12) and a $\vec{k} = (\xi, \eta, \varsigma)$ of (14) forms an energy band. If the free eigen wave function and eigen energy of the Schrödinger equation are first quantization, the three step quantization is the “second quantization”. This “second quantization” products new quantum–shout-lived quarks.

2). The rest masses and the intrinsic quantum numbers (I, S, C, B and Q) are necessary for the standard model, but they cannot be deduced by the standard model. Using (17) - (30), we deduce the rest masses and intrinsic quantum numbers of quarks from the energy bands. The deduced rest masses and quantum numbers of baryons and mesons from these masses and numbers of quarks, are consistent with experimental results. This is a strong support for the three step quantization.

3). The five quarks of the current Quark Model correspond to the five deduced ground quarks [$u \leftrightarrow u(313)$, $d \leftrightarrow d(313)$, $s \leftrightarrow d_s(493)$, $c \leftrightarrow u_c(1753)$ and $b \leftrightarrow d_b(4913)$] (see Table 11 of [10]). The current Quark Model uses only these five quarks to explain baryons and mesons. In early times, this was reasonable, natural and useful. Today, however, it is not reasonable that physicists use only these five current quarks since physicists have discovered many high energy baryons and mesons that are composed of

more high energy quarks.

4). The energy band excited quarks $u(313)$ with $\vec{n} = (0, 0, 0)$ in Table 3 and $d(313)$ with $\vec{n} = (0, 0, 0)$ in Table 4 will be short-lived quarks. They are, however, lowest energy quarks. Since there is no lower energy position that they can decay into, they are not short-lived quarks. Because they have the same rest mass and intrinsic quantum numbers as the free excited quarks $u(313)$ and $d(313)$, they cannot be distinguished from the free excited quarks $u(313)$ and $d(313)$ by experiments. The $u(313)$ and $d(313)$ with $\vec{n} = (0, 0, 0)$ will be covered up by free excited $u(313)$ and $d(313)$ in experiments. Therefore, we can omit $u(313)$ and $d(313)$ with $\vec{n} = (0, 0, 0)$ since the probability that they are produced much small than the free excited $u(313)$ -quark and $d(313)$ -quark. There are only long-lived and free excited the $u(313)$ -quark and the $d(313)$ -quark in both theory and experiments.

5). The fact that physicists have not found any free quark shows that the binding energies are very large. The baryon binding energy -3Δ (meson -2Δ) is a phenomenological approximation of the color's strong interaction energy in a baryon (a meson). The binding energy -3Δ (-2Δ) is always cancelled by the corresponding parts 3Δ of the rest masses of the three quarks in a baryon (2Δ of the quark and antiquark in a meson). Thus we can omit the binding energy -3Δ (-2Δ) and the corresponding rest mass parts 3Δ (2Δ) of the quarks when we deduce rest masses of baryons (mesons). This effect makes it appear as if there is no strong binding energy in baryons (mesons).

10 Conclusions

1). There is only one elementary quark family ϵ with three colors and two isospin states (ϵ_u with $I_Z = \frac{1}{2}$ and $Q = +\frac{2}{3}$, ϵ_d with $I_Z = -\frac{1}{2}$ and $Q = -\frac{1}{3}$) for each color. Thus there are six Fermi ($s = \frac{1}{2}$) elementary quarks with $S = C = B = 0$ in the vacuum. ϵ_u and ϵ_d have $SU(2)$ symmetries.

2). All quarks inside hadrons are the excited states of the elementary quark ϵ . There are two types of excited states: free excited states and energy band excited states. The free excited states are only the u-quark and the d-quark. They are long-lived quarks. The energy band excited states are the short-lived quarks, such as $d_s(493)$, $d_s(773)$, $u_c(1753)$ and $d_b(4913)$.

3). Since all quarks inside hadrons are excited states of the same elementary quark ϵ , all quarks ($m > 313$ Mev) will eventually decay into the $q_N(313)$ -quark [$u(313)$ and $d(313)$].

4). The three step quantization is a somewhat “second quantization” that produces the short-lived quarks.

5). There is a large binding energy -3Δ (or -2Δ) among three quarks in a baryon (or between the quark and the antiquark in a meson). It may be a possible foundation for the quark confinement.

6). We have deduced the rest masses and intrinsic quantum numbers of quarks (Table 3 and 4), baryons (Table 6) and mesons (Table 7) using the three step quantization method and phenomenological formulae. The deduced intrinsic quantum numbers of baryons and mesons match the experimental results [12] and [13] exactly, while the deduced rest masses of the baryons and the mesons are consistent with more than 98% of experimental results [12] and [13].

7). The current Quark Model is the five ground quark approximation of a more fundamental model.

8). This paper predicts some new quarks [$u_c(6073)$, $d_b(9333)$ and $d_s(773)$], baryons [$\Lambda_c(6699)$ and $B(9959)$] and mesons [$D(6231)$, $B(9502)$ and $K(3876)$].

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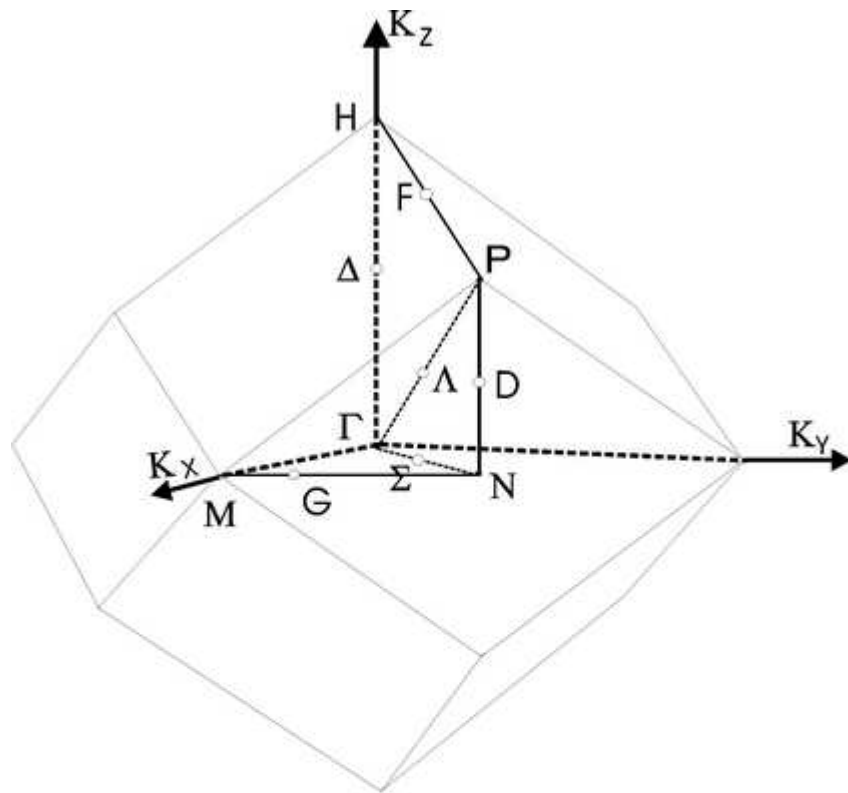
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- The axis Δ (the axis Γ -H) is a four fold rotation axis
- ▲ The axis Λ (the axis Γ -P) is a three fold rotation axis
- ▮ The axis Σ (the axis Γ -N) is a two fold rotation axis

Figure 1

Figure 1: The regular rhombic dodecahedron. The symmetry points and axes are indicated.